Ex Evaluate $\iint_R \sin(9x^2+4y^2)\,dA$ where R is the region in the first quadrant bounded by the ellipse $9x^2+4y^2=1$.

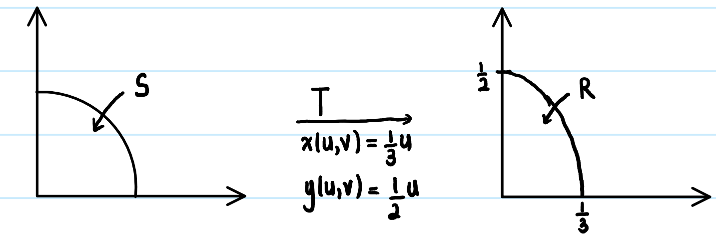
Ans Now we want to make the appropriate change of variables.

Let $u = 3x$ and $v = 2y$

Then $9x^2+4y^2 = u^2+v^2$

$$x = \frac{1}{3}u,\quad y = \frac{1}{2}v \quad \Rightarrow \quad \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1/3 & 0 \\ 0 & 1/2 \end{vmatrix} = \frac{1}{6}$$

and R is the image of the quarter disc given by $u^2+v^2 \le 1$, $u \ge 0, v \ge 0$.



Then, $$\iint_R \sin(9x^2+y^2)\,dA = \iint_S \frac{1}{6}\sin(u^2+v^2)\,du\,dv$$

But now we can solve the integral over S using polar coordinates (i.e make another substitution)

$$\iint_S \frac{1}{6}\sin(u^2+v^2)\,du\,dv = \int_0^{\pi/2}\int_0^1 \frac{1}{6}\sin(r^2)\,r\,dr\,d\theta = \frac{1}{6}\int_0^{\pi/2} d\theta \int_0^1 r\sin(r^2)\,dr = \frac{\pi}{12}\left[-\frac{1}{2}\cos(r^2)\right]_0^1 = \frac{\pi}{24}(1-\cos 1)$$